PEOPLES´ FRIENDSHIP UNIVERSITY OF RUSSIA NAMED AFTER PATRICE LUMUMBA

RUDN UNIVERSITY

Tests for RUDN University Open Olympiad for Foreign Citizens

MATHEMATICS (M)

Variant 1

Choose one or more correct answers in the test tasks.

Transfer the answer numbers to the answer sheet:

question	answers	question	answers
1		16	
2		17	
3		18	
4		19	
5		20	
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

1.Find the solution to the limit: $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\mathrm{tg}^3(x)-3\mathrm{tg}(x)}{\cos\left(x+\dfrac{\pi}{6}\right)}$

a. $x=-8$
b. $x=0$
c. $x=-24$
d. $x=16$

2. Which of the proposed values is the solution to the derivative of the function: $\log_7(\arccos(3x))$

1) $-\dfrac{3}{\arccos(3x)\sqrt{1-9x^2}\ln(7)}$

2) $\dfrac{3}{\ln(7)\arccos(3x)\sqrt{1+9x^2}}$

3) $\dfrac{3}{\ln(7)\sqrt{1-9x^2}\arccos(3x)}$

4) $-\dfrac{3}{\sqrt{1+9x^2}\arccos(3x)\ln(7)}$

a. 1
b. 2
c. 3
d. 4

3. Which of the proposed values is the solution to the indefinite integral: $I=\int x^2\sin(2x)dx$

1) $\dfrac{x\cos(2x)}{2}+\dfrac{x^2\sin(2x)}{2}-\dfrac{\sin(2x)}{4}+C, C=const$

2) $\dfrac{x\sin(2x)}{2}-\dfrac{x^2\cos(2x)}{2}+\dfrac{\cos(2x)}{4}+C, C=const$

3) $\dfrac{x\sin(2x)}{2}+\dfrac{x^2\cos(2x)}{2}+\dfrac{\sin(2x)}{4}+C, C=const$

4) $-\dfrac{x\cos(2x)}{2}+\dfrac{\sin(2x)}{4}-\dfrac{x^2\sin(2x)}{2}+C, C=const$

a. 1
b. 2
c. 3
d. 4

4. Which of the proposed values is the solution to the definite integral: $I=\int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{2}}\cos(x)\cos(2x)\cos(3x)dx$

1) $\dfrac{5\pi}{24}+\dfrac{3\sqrt{3}}{32}$

2) $-\dfrac{11}{96}$

3) $\dfrac{5}{96}$

4) $\dfrac{5\pi}{24}+\dfrac{\sqrt{3}}{32}$

a. 1

b. 2
c. 3
d. 4

5.**Which of the proposed values is the solution to the product of matrices**: $\begin{pmatrix} -2 & 1.6 & 8 \\ 9 & -5 & 3.4 \\ 2.4 & -6 & -4 \end{pmatrix}$ and $\begin{pmatrix} -8 & 3 & -3.1 \\ 2 & 2.6 & -9 \\ 1.5 & -5 & 3 \end{pmatrix}$

1) $\begin{pmatrix} 31.2 & -41.84 & 15.8 \\ -56.9 & -3 & -62.7 \\ -37.2 & -28.4 & 34.56 \end{pmatrix}$

2) $\begin{pmatrix} 31.2 & -41.84 & 15.8 \\ -76.9 & -3 & 27.3 \\ -37.2 & 11.6 & 34.56 \end{pmatrix}$

3) $\begin{pmatrix} 31.2 & -41.84 & 15.8 \\ -24.8 & -3 & -27.3 \\ -29.1 & 11.6 & 34.56 \end{pmatrix}$

4) $\begin{pmatrix} 31.2 & -41.84 & 15.8 \\ -76.9 & -27 & -62.7 \\ -33.2 & -28.4 & 34.56 \end{pmatrix}$

a. 1
b. 2
c. 3
d. 4

6.Find minor M_{34} **of matrix** $A = \begin{pmatrix} 8 & 5 & 0 & -7 & -2 \\ 2 & -3 & 8 & 6 & 9 \\ -1 & 0 & -4 & 7 & 1 \\ 4 & 2 & 3 & 0 & -8 \\ -6 & -5 & 0 & -7 & 6 \end{pmatrix}$

a. -750
b. 2520
c. -1482
d. 530

7. Find the value of the expression $2\vec{a} - 3\vec{b}$ **if** $\vec{a} = \{17.3, -4.8, 3.5\}$ **and** $\vec{b} = \{-5.9, 11.6, -4.1\}$

a. $\{52.3, -21.5, -5.3\}$
b. $\{16.9, -37.6, 18.7\}$
c. $\{52.3, -44.4, 19.3\}$

d. {16.9, 25.2, 2.3}

8. Which of the proposed values is the solution to the distance between the points: A(4.6, -2, 0.7) and B(3, 1.2, -4)

1) $AB = \sqrt{34.89}$
2) $AB = \sqrt{90.09}$
3) $AB = \sqrt{23.69}$
4) $AB = \sqrt{14.09}$

a. 1
b. 2
c. 3
d. 4

9. Which of the proposed values is the solution to the equation: $y'y^2\sqrt{y} = \frac{1}{\sin^2(x)}$

1) $y = \sqrt[7]{\frac{49}{4}\left(C + \mathrm{tg}(x)\right)^2}, C = const$
2) $y = \sqrt[3]{\frac{49}{4}\left(C - \mathrm{ctg}(x)\right)^2}, C = const$
3) $y = \sqrt[3]{\frac{49}{4}\left(C + \mathrm{tg}(x)\right)^2}, C = const$
4) $y = \sqrt[7]{\frac{49}{4}\left(C - \mathrm{ctg}(x)\right)^2}, C = const$

a. 1
b. 2
c. 3
d. 4

10. Which of the proposed values is the solution to the equation: $x^2y' = y(x - y)$

1) $y = -\frac{x}{\ln(x) + C}, C = const$
2) $y = \frac{-x}{\ln|x| + C}, \; y = 0, \; C = const$
3) $y = \frac{1}{\ln|x| + C}, \; C = const$
4) $y = \frac{x}{\ln|x| + C}, \; y = 0, \; C = const$

a. 1

b. 2
c. 3
d. 4

11. Which of the proposed values is the calculation of the l_3-norm of vector $\vec{a} = (3, 5, -4, 1, -2)$

1) $\sqrt[3]{15}$
2) $\sqrt[3]{225}$
3) $\sqrt{15}$
4) $3\sqrt[3]{3}$

12. Which method of numerical solution of equations is based on the following corollary of the Bolzano-Cauchy theorem: Let $f(x) \in C([a,b])$ be the continuous function then if $sign(f(a)) \neq sign(f(b))$ then $\exists c \in [a,b]: f(c) = 0$

a. Bisection method
b. Newton's method
c. Secant method
d. Tangent method

13. Which of the suggested values is the formula for midpoint rectangles:

1) $\int_a^b f(x)dx \approx h\sum_{i=1}^{n} f\left(x_{i-1} + \frac{h}{n}\right) = h\sum_{i=1}^{n} f\left(x_i - \frac{h}{n}\right),\ h = \frac{b-a}{n}$
2) $\int_a^b f(x)dx \approx h\sum_{i=1}^{n} f\left(x_{i-1} + \frac{h}{2}\right) = h\sum_{i=1}^{n} f\left(x_i - \frac{h}{2}\right),\ h = \frac{b-a}{2}$
3) $\int_a^b f(x)dx \approx h\sum_{i=1}^{n} f\left(x_{i-1} + \frac{h}{2}\right) = h\sum_{i=1}^{n} f\left(x_i - \frac{h}{2}\right),\ h = \frac{a+b}{n}$
4) $\int_a^b f(x)dx \approx h\sum_{i=1}^{n} f\left(x_{i-1} + \frac{h}{2}\right) = h\sum_{i=1}^{n} f\left(x_i - \frac{h}{2}\right),\ h = \frac{b-a}{n}$

a. 1
b. 2
c. 3
d. 4

14. 7 boys and 2 girls are seated in random order at a round table with 9 chairs. Find the probability that the girls will not sit next to each other.

a. 0.75
b. 0.55
c. 0.25
d. 0.2

15. The pencil case contains 7 green, 4 yellow and 2 orange pencils. Three pencils are taken out in succession. Find the probability that the first pencil will be orange, the second will be green, and the third will be yellow.

1) $\frac{56}{2197}$
2) $\frac{14}{429}$
3) $\frac{1889}{1716}$
4) $\frac{13}{56}$

a. 1
b. 2
c. 3
d. 4

16. A company produces headphones. The company's first plant produces 15% of all headphones, the second plant produces 45% and the third plant makes 40%. The defect is 8%, 10%, 3% respectively. The released headphones turned out to be defective. Find the probability that they were manufactured in the third plant.

1) $\frac{3}{25}$
2) $\frac{69}{1000}$
3) $\frac{4}{23}$
4) $\frac{3}{40}$

a. 1
b. 2
c. 3
d. 4

17. A random variable is given by a distribution series:

$\xi \setminus \eta$	-2	1	2
1	0.2	0	0.3
2	0.1	0.2	0.2

Find the mathematical expectation of the random variable $\mu = \xi \cdot \log_2 |\eta|$

a. 0.8
b. 6
c. 4.4
d. 1.1

18. The basket contains quite a lot of balls: red, purple, white and green. In how many ways can three balls be taken out of the basket?

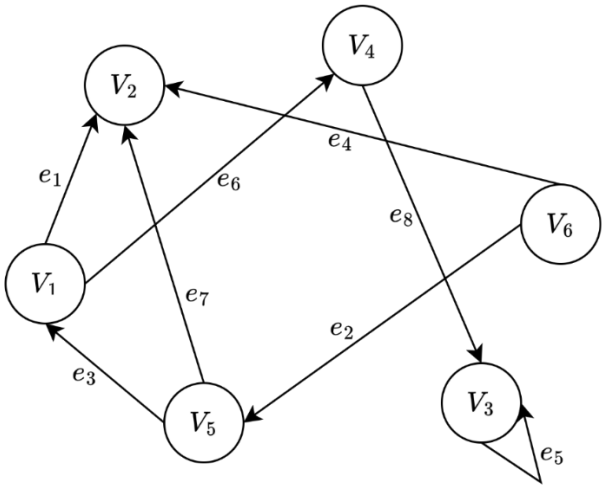
a. 4
b. 24
c. 20
d. 64

19. Which of the proposed values is the term of Newton binomial expansion $\left(x^3 - 2\sqrt{5}\right)^9$ containing x^{18}

1) $-3360\sqrt{5}$
2) $672\sqrt{5}$
3) $-20160\sqrt{5}$
4) $3360\sqrt{5}$

a. 1
b. 2
c. 3
d. 4

20. Which of the proposed values is the correct incidence matrix for the undirected graph:



1) $\begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

2) $\begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

3) $\begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

4) $\begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

a. 1
b. 2
c. 3
d. 4